

On the Maximum Efficiency of the Ideal Regenerative Stirling Cycle

R. M. Abrahamian

State Engineering University of Armenia, Yerevan, Armenia

Received September 23, 2009

Abstract—It is shown that the efficiency of a regenerative Stirling cycle can principally not reach that of a Carnot cycle operating in the same temperature range. A relation is obtained between the degree of regeneration and the characteristics of regenerator. The maximum efficiency of an ideal regenerative Stirling engine is obtained.

DOI: 10.3103/S1068337210030072

Key words: Stirling cycle, regeneration, efficiency, Carnot cycle

1. INTRODUCTION

Stirling engines, which combine affordability of Diesel engines, lightness of carburetor engines, unpretentiousness to fuel and dependability of internal-combustion engines, attract at present high attention. In addition, Stirling engines are rather noiseless. Revival of the old concept owes new materials, new construction, and ecological requirements, due to which Stirling engines find application in a variety of different fields of technology, from cardiac muscles to nuclear ships and spacecrafts [1–3].

In the thermodynamic theory of heat engines, it is accepted to consider as standard theoretical cycles characterized by reversibility of all processes and constancy of physical properties of working substance. An ideal Stirling cycle, consisting of two isotherms and two isochors, has thermal efficiency determined by the temperature interval between the heater and the cooler, $T_1 - T_2$, and the compression degree $r = V_1/V_2$, as shown in the figure [1–3]:

$$\eta = \frac{T_1 - T_2}{T_1 + (T_1 - T_2)/(\gamma - 1) \ln r}. \quad (1)$$

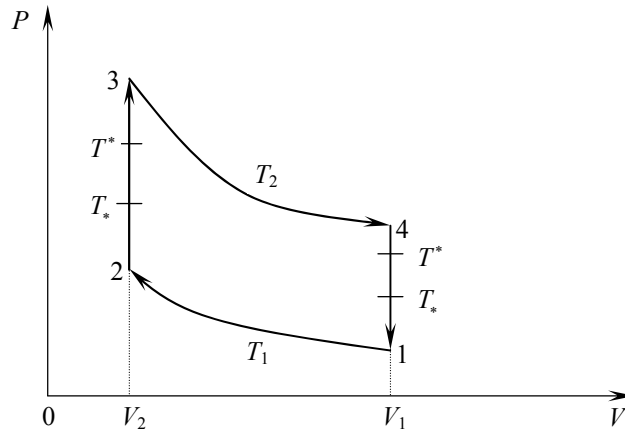
Here γ is the adiabatic index and the working substance is an ideal gas.

One of basic units of Stirling engine is the regenerator, which operates by the principle of heat accumulation, reducing fuel consumption and raising the system efficiency. At the same time the regenerator takes the function of the cooler by assisting working substance to go back to the initial, cooler, state. In this case the cycle efficiency is determined by the formula (see, e.g., [1, 4])

$$\eta = \frac{T_1 - T_2}{T_1 + [(T_1 - T_2)/(\gamma - 1) \ln r](1 - \beta)}, \quad (2)$$

where the dimensionless quantity β (the degree of regeneration) is always smaller than unity.

In literature on Stirling cycles and engines it is often stated that the theoretical efficiency of an ideal Stirling cycle with regenerator can equal the efficiency of the Carnot cycle operating in the same temperature interval. Such a belief is a consequence of the assumption that the regeneration degree β can take values arbitrarily close to unity. So, e.g., work [4] for estimation of the Stirling-engine efficiency assumed that β can acquire values of 0.8–0.9. Note that at $\beta = 1$ the efficiency of the Stirling cycle equals that of the Carnot cycle, but the values of the degree of regeneration given in [4] are obtained from consideration of processes in a counter flow heat exchanger, rather than operation of piston heat engines with external heat supply, which is the Stirling engine. Such “idealization” of the ideal Stirling cycle may hinder realistic estimation of the role of Stirling cycle in the hierarchy of thermodynamic cycles employed for construction of heat engines. In the present work it will be shown that the degree of regeneration β of the Stirling engine cannot be larger than 0.5.



Ideal Stirling cycle. T^* and T_* are the steady-state temperatures of the regenerator.

2. THEORETICAL PART

Let the mass of the working substance be m_0 and its constant-volume specific heat C_V be independent of temperature. The mass of regenerator is m and the specific heat of its material is C . It should be noted that the regenerator must have good heat conduction and large specific heat, which is not always compatible. Let the quantity $\alpha = mC/m_0C_V$ be termed characteristic of regenerator. Let the temperature of regenerator vary from T_1 to T_2 . If the Stirling engine is launched with cool regenerator at temperature T_2 , the heat balance equations provide the quantity of heat transferred at heating from regenerator to working substance: $Q = m_0C_V(T' - T_2)$, where $T' = (\alpha T_1 + (1 + \alpha + \alpha^2)T_2)/(1 + \alpha)^2$. By expressing $T' - T_2$ in terms of $T_1 - T_2$ it is obtained $T' - T_2 = \alpha(T_1 - T_2)/(1 + \alpha)^2 = \beta_1(T_1 - T_2)$, where β is the degree of regeneration in the initial cycle. As seen in formula (2), the highest efficiency is obtained at the highest value of β . Recall that the degree of regeneration β is always positive and smaller than unity. It is obvious that for the first regenerative cycle of the Stirling engine optimum value of α is unity and the degree of regeneration β takes the maximum value 0.25. In subsequent cycles optimum values of β and α will increase, but the α dependence of β will be different. For the second cycle it will be $\beta_2 = (\alpha + 2\alpha^2 + 2\alpha^3)/(1 + \alpha)^4$, while for the third cycle $\beta_3 = (\alpha + 4\alpha^2 + 7\alpha^3 + 6\alpha^4 + 3\alpha^5)/(1 + \alpha)^6$. Similarly the optimum values are obtained to be $\alpha_2 = 1.54$ at $\beta_2 = 0.32$, and $\alpha_3 = 2.03$ at $\beta_3 = 0.364$.

So, the temperature of regenerator and the degree of regeneration will increase until steady state will be reached. In the steady state regenerator will complete a peculiar "cycle", heating in the process 4 – 1 from temperature T_* to T^* at the expense of cooling of working substance and then giving heat back to working substance in the process 2 – 3 being cooled by the same amount (see the figure). In the first process the heat balance equation gives

$$T^* - T_1 + \alpha(T^* - T_*) = 0. \quad (3)$$

At transferring the heat from regenerator to working substance one has

$$T_* - T_2 + \alpha(T_* - T^*) = 0. \quad (4)$$

So, regenerator reduces the consumption of fuel from the external source by $Q = m_0C_V(T_* - T_2)$.

The heat exchange between the working substance and regenerator was assumed to be perfect, i.e., without losses. From formulas (3) and (4) it follows that

$$T_* = \frac{\alpha T_1 + (\alpha + 1)T_2}{2\alpha + 1}. \quad (5)$$

For the quantity of heat it is obtained

$$Q = m_0C_V(T_1 - T_2) \frac{\alpha}{2\alpha + 1} = m_0C_V(T_1 - T_2)\beta, \quad (6)$$

where $\beta = \alpha/(2\alpha + 1)$ is the degree of regeneration in steady regime.

For determination of the efficiency of the Stirling cycle with allowance for the action of regenerator one has the formula

$$\eta = \frac{A_{12} + A_{34}}{Q_1 + Q'_2 - Q}. \quad (7)$$

Here A_{12} is the work of the ideal gas during isothermic compression, i.e.,

$$A_{12} = -\frac{m_0}{\mu} RT_2 \ln r, \quad (8)$$

while A_{34} is the same work, but during isothermic expansion:

$$A_{34} = \frac{m_0}{\mu} RT_1 \ln r. \quad (9)$$

The quantity $Q_1 = Q_{23} = m_0 C_V (T_1 - T_2)$ is the heat received by the working substance from the heater in the process 2–3 and $Q'_1 = Q_{34} = (m_0/\mu) RT_1 \ln r$ is the heat received by the working substance during isothermic expansion.

Substituting these data into formula (7) results in

$$\eta = \frac{\frac{m_0}{\mu} R(T_1 - T_2) \ln r}{\frac{m_0}{\mu} RT_1 \ln r + m_0 C_V (T_1 - T_2)(1 - \beta)}. \quad (10)$$

It is easy to see that expression (10) coincides with formula (2), but with β defined by formula (6). Since the value of α is always positive, β may vary from zero to 0.5. Note that the value $\beta = 0.5$ corresponds to infinitely large mass of the regenerator. In this case the maximum efficiency of an ideal Stirling cycle is obtained to be $\eta = (T_1 - T_2)/(T_1 + 0.5(T_1 - T_2)/(\gamma - 1) \ln r)$.

3. DISCUSSION

Let some numerical estimations be given for the efficiency of the Stirling cycle. For helium as a working substance, $\gamma = 1.67$, and $r = 5$, if $T_1 = 400$ K and $T_2 = 300$ K (the efficiency of the Carnot cycle is in this case 0.25), it is obtained 0.203 without regenerator and 0.214 with regenerator. If $T_1 = 750$ K and $T_2 = 400$ K, the Carnot cycle has the efficiency 0.467, the Stirling cycle without regenerator 0.325, and the Stirling cycle with regenerator 0.383. If the working substance is air with the regenerator of steel, it is obtained $\beta = 0.4$ ($\alpha = 2$) and $m/m_0 = 0.97$. In more realistic conditions, namely with allowance for heat losses and gas imperfectness, the efficiency will be changed.

ACKNOWLEDGMENT

In conclusion, I consider it a pleasant duty to express my gratitude to Professor N.B. Engibaryan for formulation of the problem and valuable discussions.

REFERENCES

1. Rider, G.T. and Huper, C., *Stirling Engines*, London–New York: E. and F. N. Spon, 1983.
2. Urielli, I. and Berchowitz, D.M., *Stirling Cycle Engine Analysis*, Bristol: Taylor & Francis Group, 1984.
3. Organ, A., *The Regenerator and the Stirling Engine*, New York: John Wiley & Sons, 1997.
4. Bundin, A.A., *Izvestiya Vuzov, Mashinostroyeniye*, 1969, no. 12, p. 106.